

# Our Crew at VBLHEP



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# Puzzles of multiplicity

International Student Practice  
Joint Institute for Nuclear Research

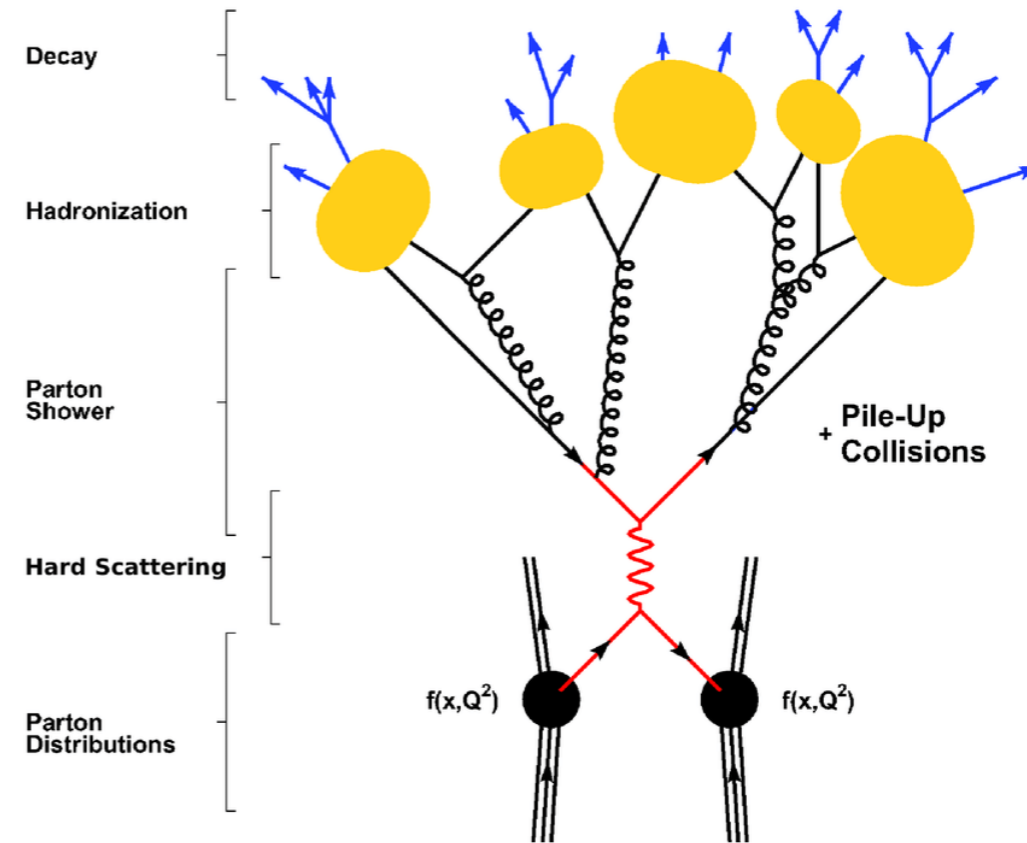


Klaudia Sajdaková  
Lukáš Tropp

Dubna, 26. 7. 2019



# Puzzles of multiplicity



Bauer, Julia & Muller, Thomas. (2019). Prospects for the Observation of Electroweak Top Quark Production with the CMS Experiment.

**Multiplicity** - number of created secondary particles

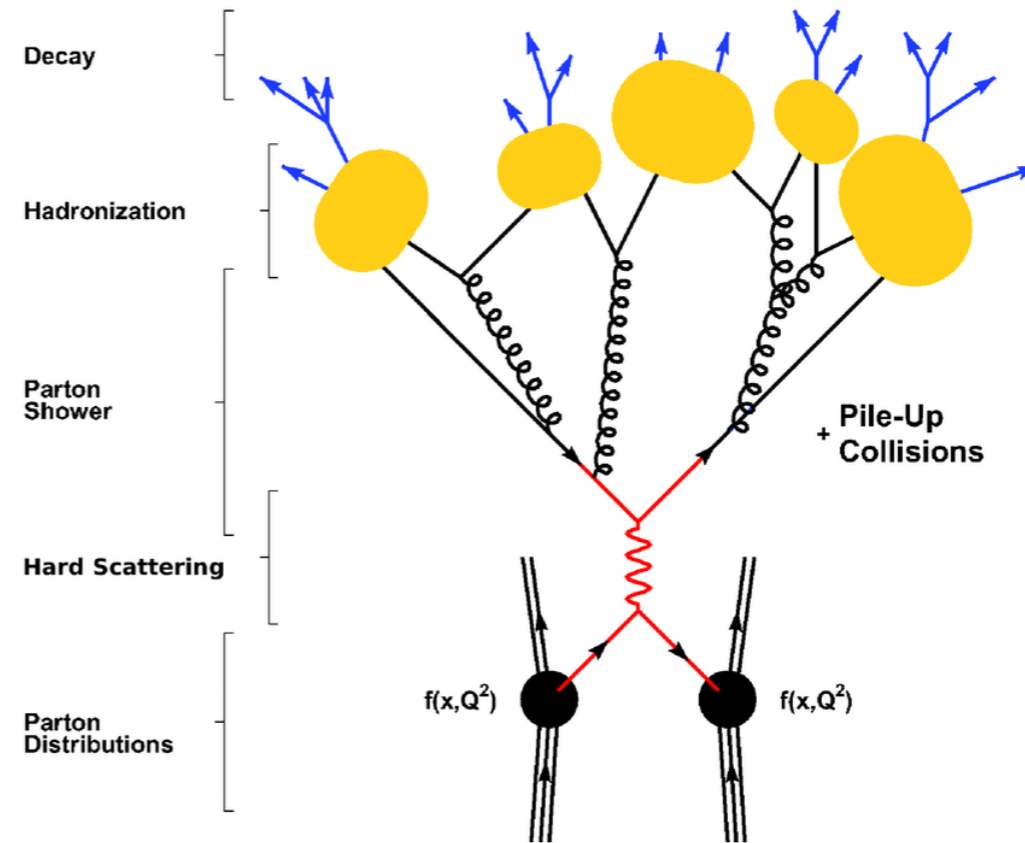
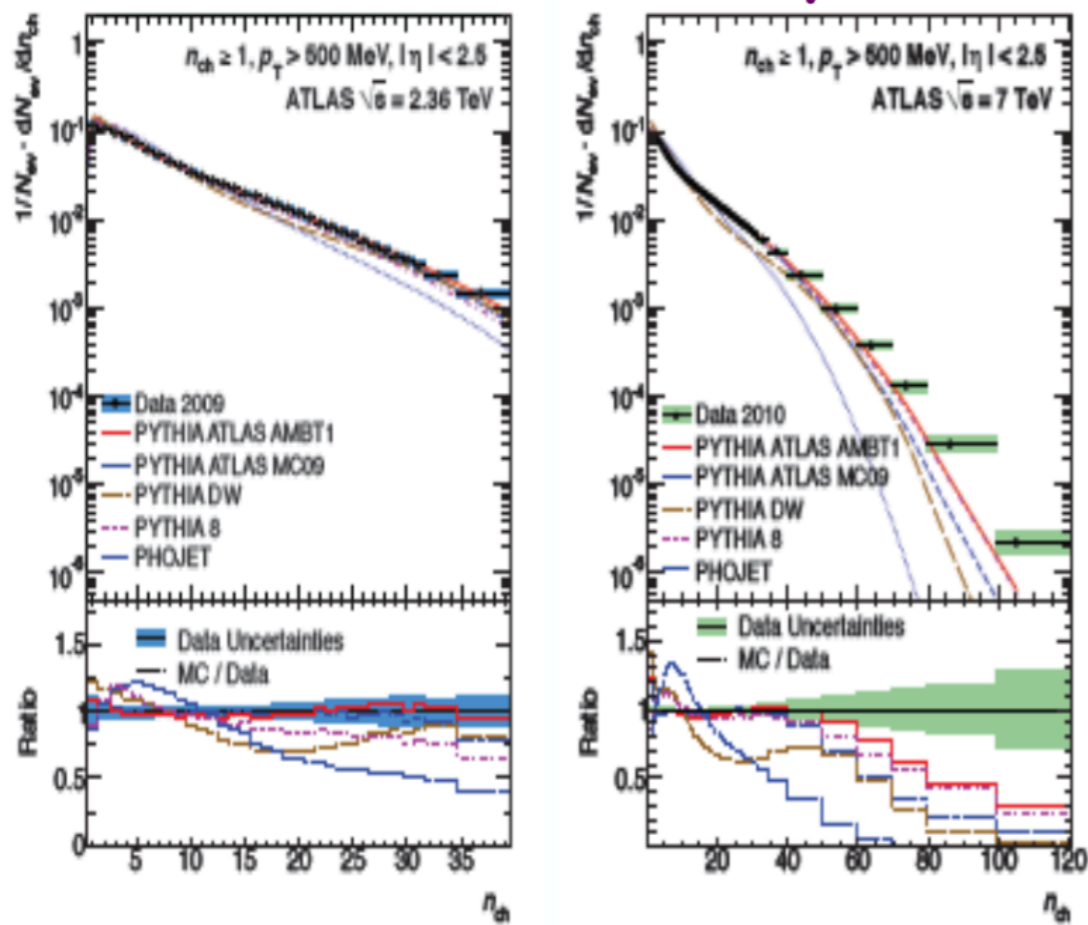
**High Multiplicity (HM) events** - connected with collective behaviour (ridges, flow, shock waves etc.)

# Puzzles of multiplicity

**Hadronization** - not fully understood process

**Model vs. Data** - we have observed discrepancies for high multiplicity events

**ATLAS Coll. A. Morley, 2015**



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# Puzzles of multiplicity

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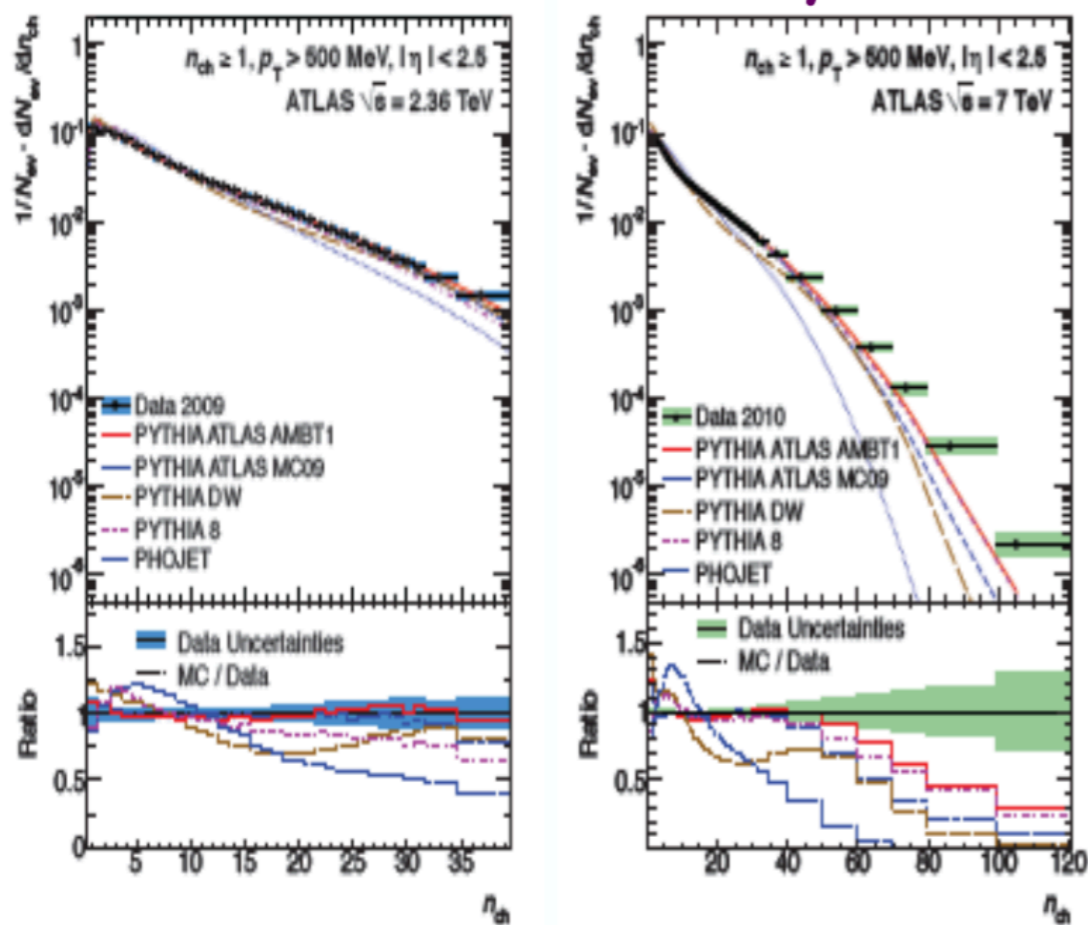
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Will be presented

Multiparticle production in :

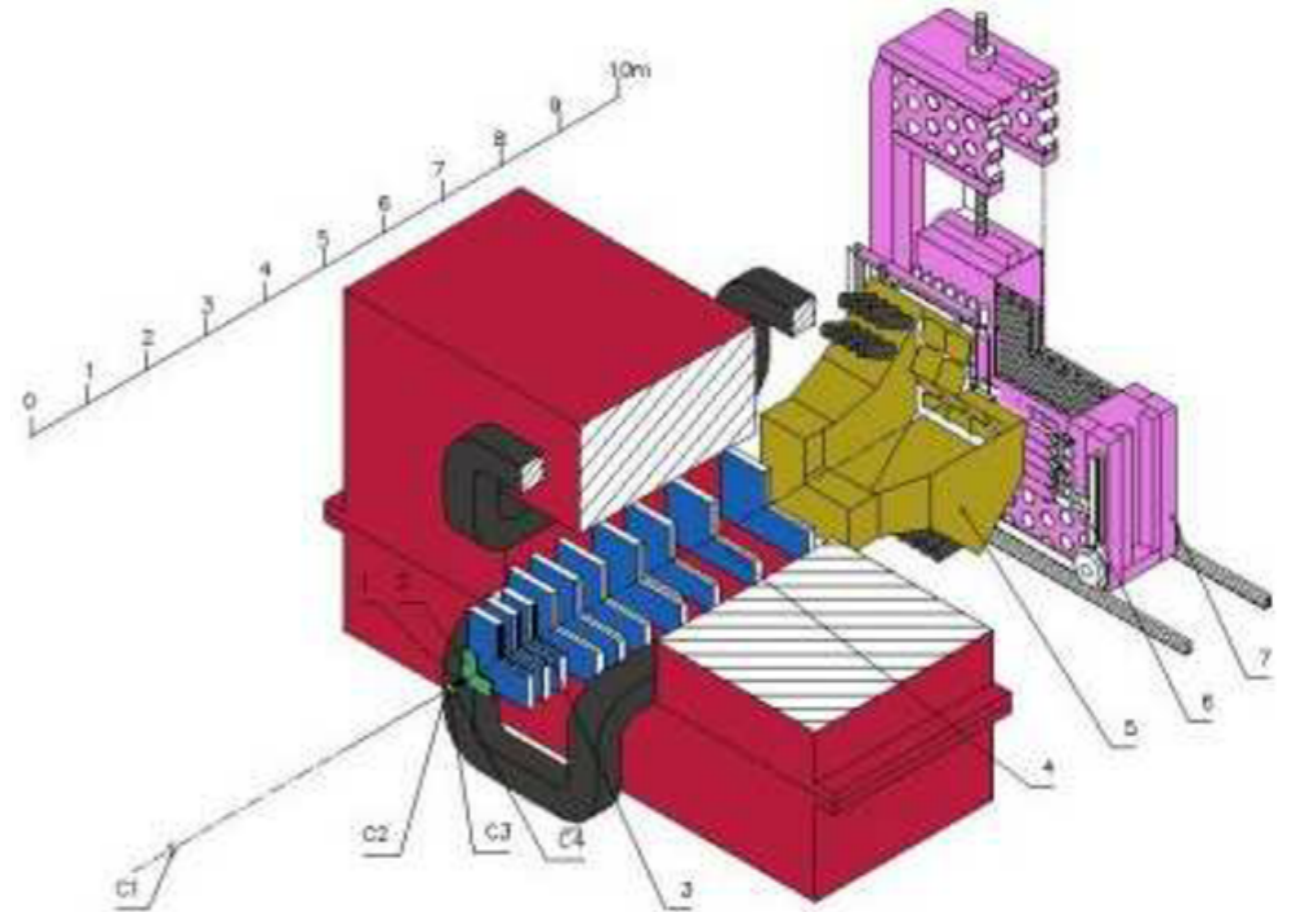
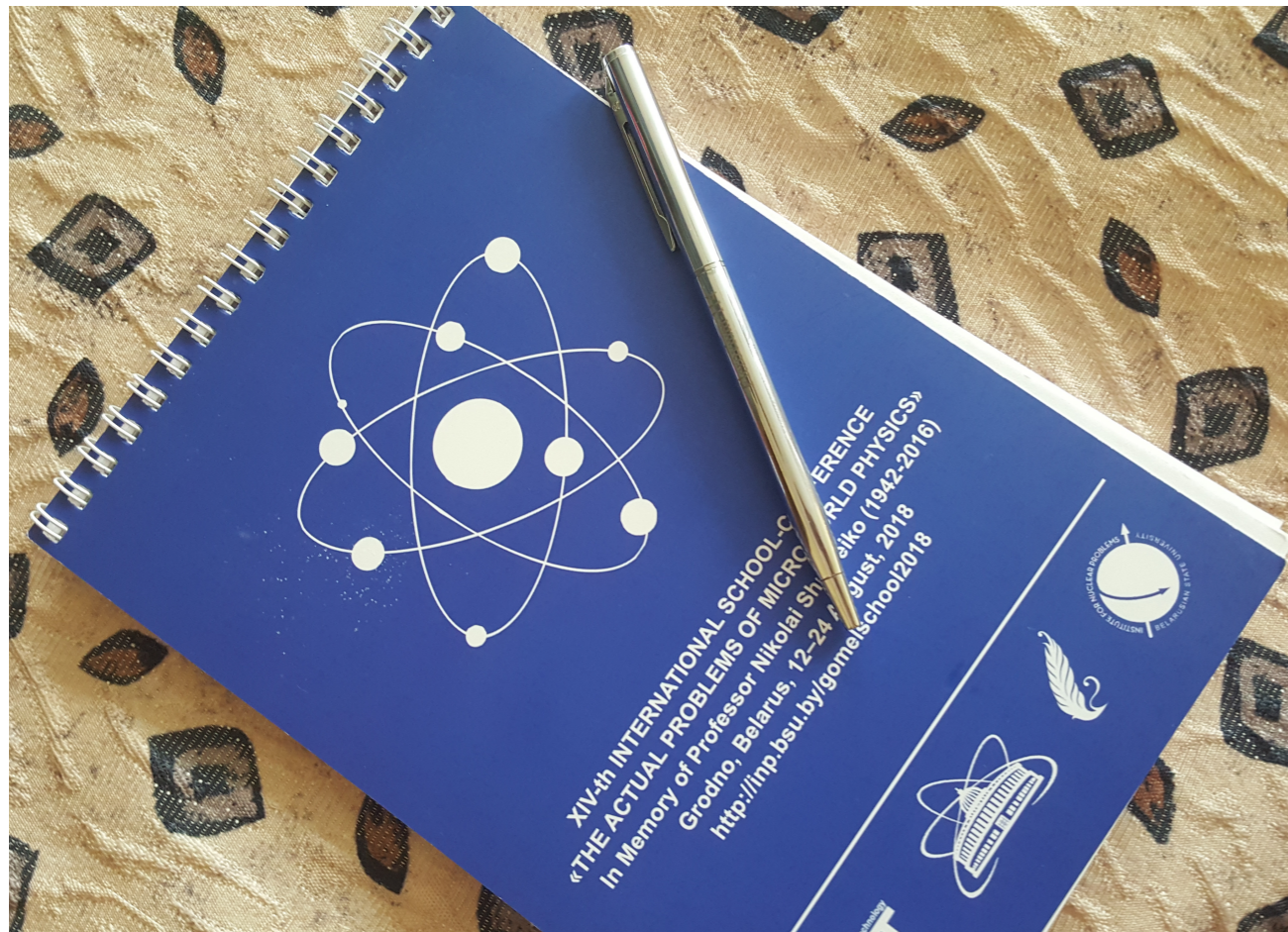
1. e-e<sup>+</sup> annihilation
2. quarkonia decay
3. pp interactions
4. p(anti)p annihilation

ATLAS Coll. A. Morley, 2015



# Toolkit

## SVD2 collaboration



**Spectrometer with Vertex Detector**



# Description of particle production Two Stage Model (STM)

Instead of hard-working with multiplicity distribution (MD) we use **generating function (GF)**

How to get multiplicity distribution from **GF**

$$G(z) = \sum_n P_n z^n$$

$$P_n = \frac{1}{n!} \frac{\partial^n}{\partial z^n} G(z) \Big|_{z=0}$$

For cumulants we get

$$F_1 = G'(z) \Big|_{z=1} = \sum_n P_n n z^{n-1} \Big|_{z=1} = \bar{n}$$

$$F_2 = G''(z) \Big|_{z=1} = \overline{n(n-1)} = \overline{n^2} - \bar{n}$$

}

**second correlative moment**

$$f_2 = G'' - (G')^2 = F_2 - F_1^2$$

Poisson distribution (PD) }  
 $f_2 = 0$

**if  $f_2 = 0$  - independent process of formation**

Binomial distribution (BD)

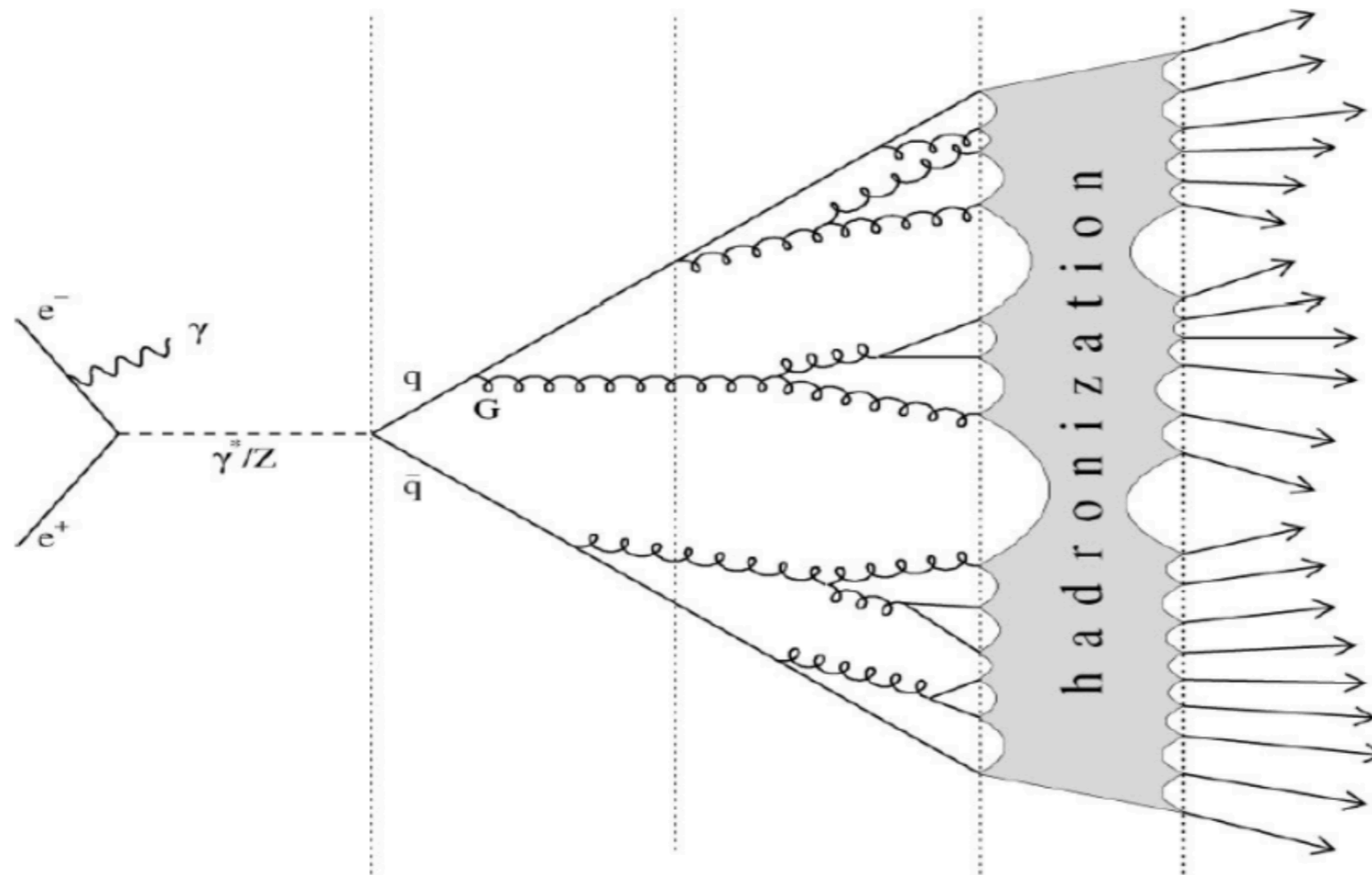
$$f_2 < 0$$

Negative Binomial distribution (NGB)

$$f_2 > 0$$

# $e^+e^-$ annihilation

$$e^+e^- \rightarrow \gamma(Z^0) \rightarrow (q, g) \rightarrow ? \rightarrow \text{hadrons}$$



**1. stage**

qg-cascade

**2. stage**

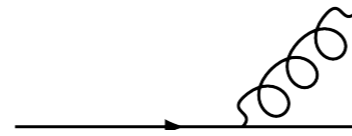
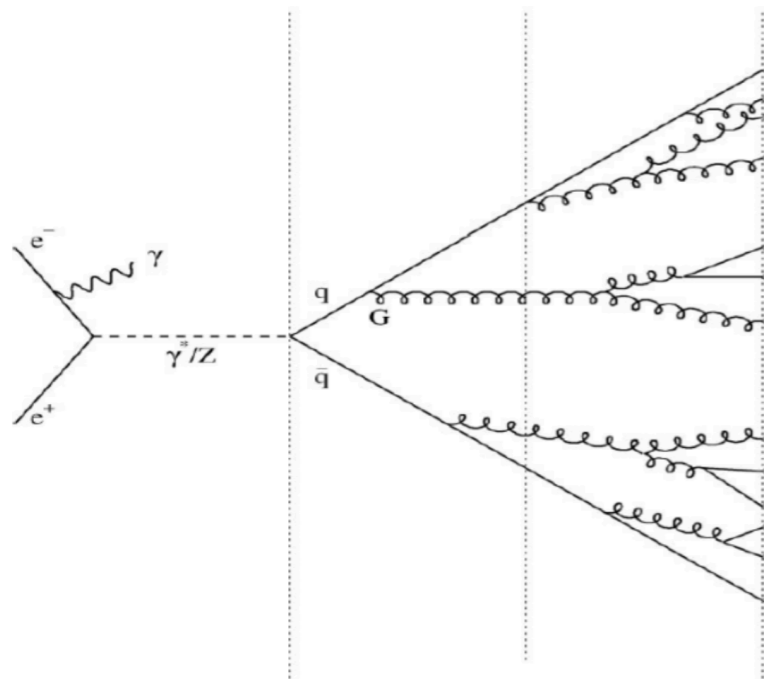
hadronization



# $e^+e^-$ annihilation - I. stage

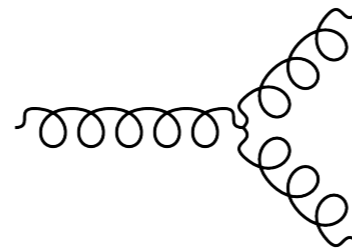
**qq-cascade** is based on pQCD

Three elementary processes :



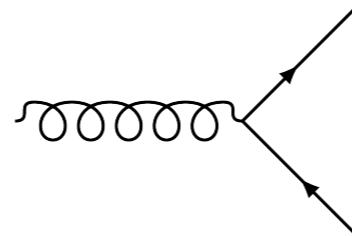
**bremsstrahlung**

probability A



**gluon splitting**

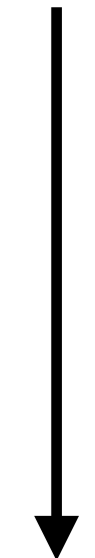
probability  $\tilde{A}$



**quark-(anti)quark  
pair creation**

probability B

High

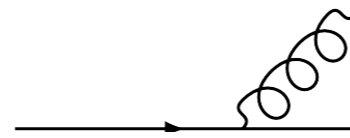
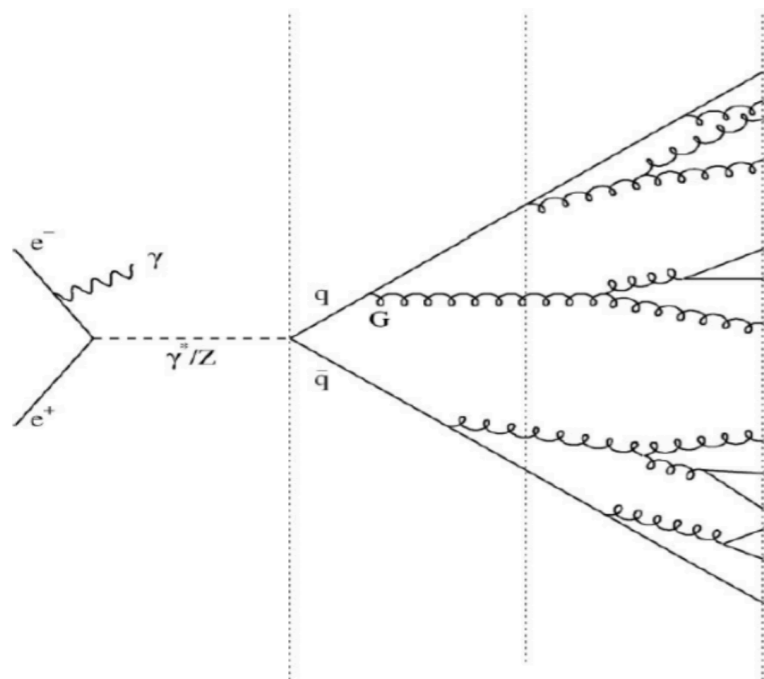


Low

# $e^+e^-$ annihilation - I. stage

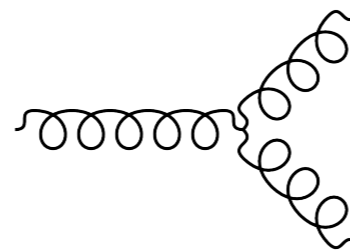
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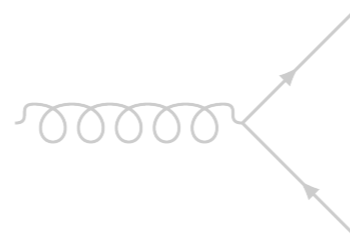
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probability A



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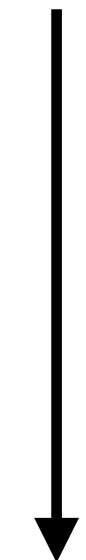
probability  $\tilde{A}$



quark-(anti)quark  
pair creation

probability B

High



Low

**gluon jet (Furry)**

$$P_m^g = \frac{1}{\bar{m}} \left( 1 - \frac{1}{\bar{m}} \right)^{m-1}$$

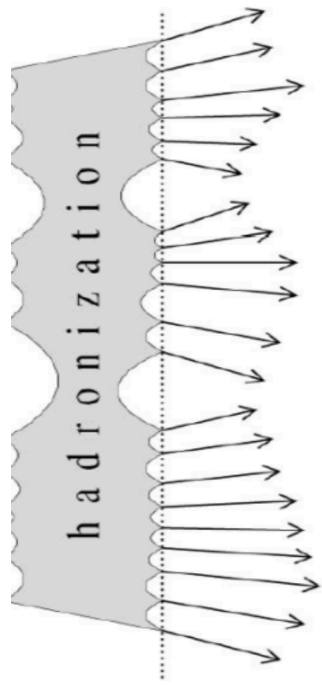
$$k_p = \frac{A}{\tilde{A}}$$

MD

**quark jet (NBD)**

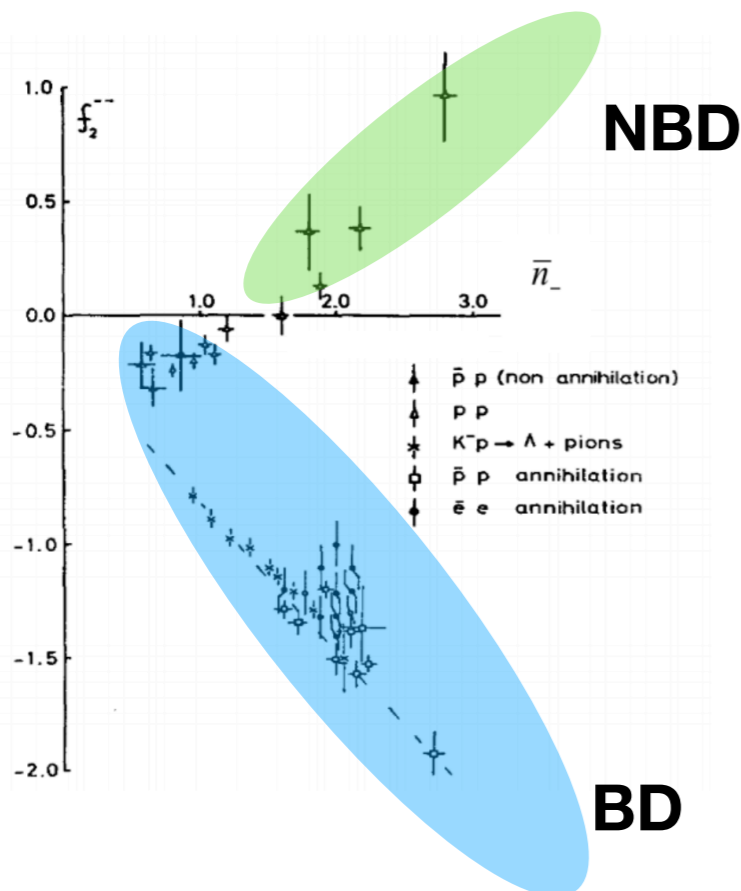
$$P_m^q = \frac{k_p(k_p + 1) \dots (k_p + m - 1)}{m!} \left( \frac{\bar{m}}{\bar{m} + k_p} \right)^m \left( \frac{k_p}{\bar{m} + k_p} \right)^{k_p}$$

# e<sup>+</sup>e<sup>-</sup> annihilation - II. stage



At the low energy region the contribution of hadronization is predominant => we choose BD (at low energy  $f_2 < 0$ )

$$P_n^H = C_{N_p}^n \left( \frac{\bar{n}_p^h}{N_p} \right)^n \left( 1 - \frac{\bar{n}_p^h}{N_p} \right)^{N_p - n}$$



$\bar{n}_p^h$

mean multiplicity

$N_p$

maximum number of hadrons formed from single parton at its passing through hadronization

# e<sup>+</sup>e<sup>-</sup> annihilation

## Convolution of two stages

is based on :

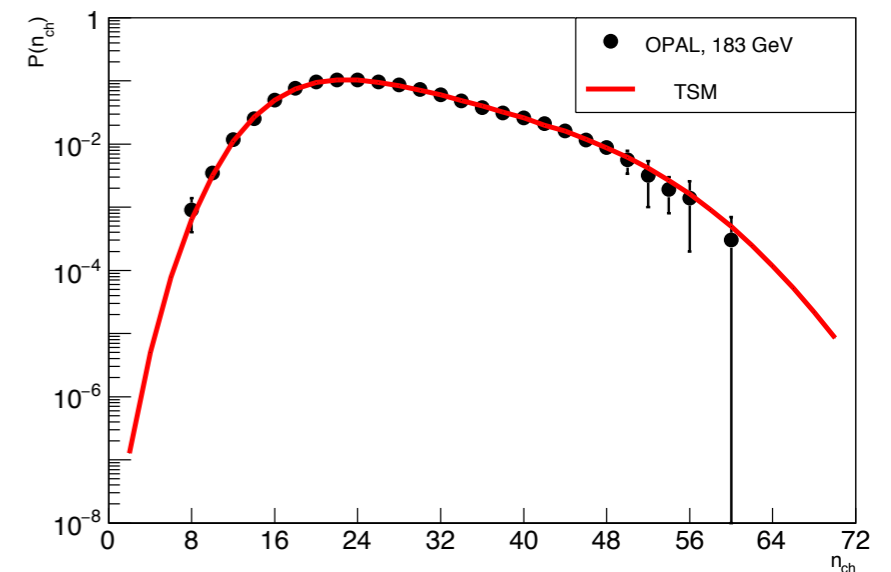
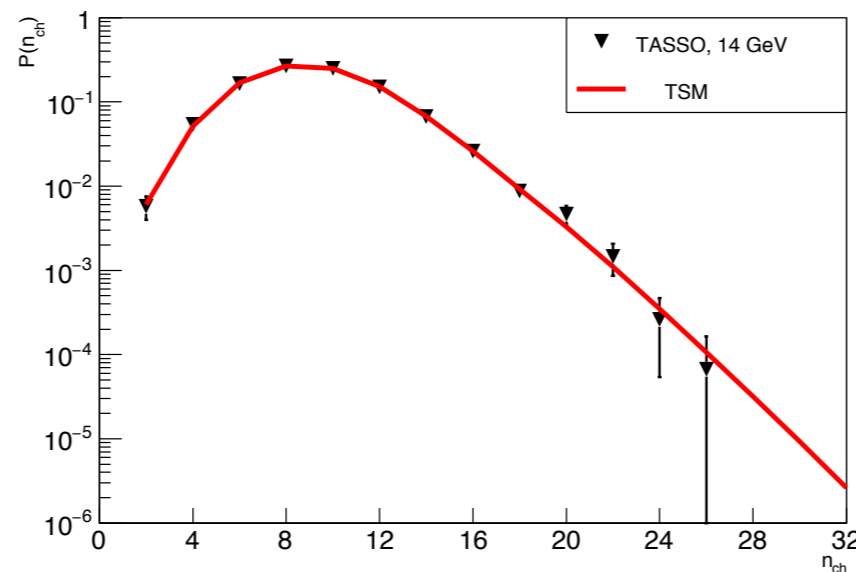
- soft dicolouration
- equality of the hadron production probabilities from quark and gluon at the second stage

$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m^P C_{(2+\alpha m)N}^n \left( \frac{\bar{n}^h}{N} \right)^n \left( 1 - \frac{\bar{n}^h}{N} \right)^{(2+\alpha m)N-n}$$

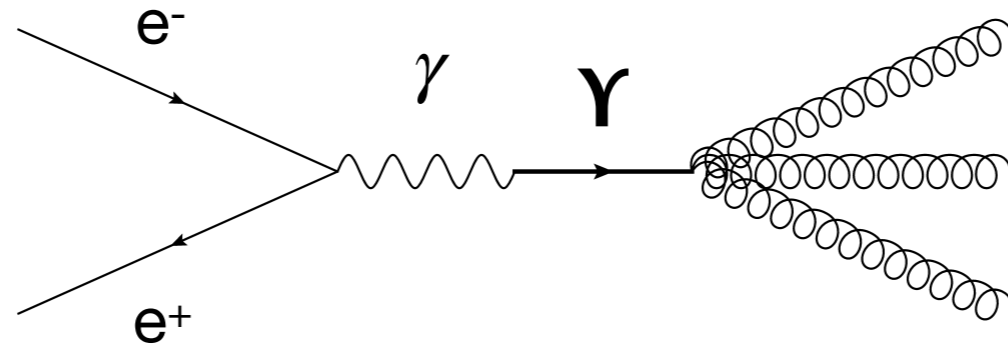
$\Omega$       normalization factor

$M_g$       number of active gluons

### Data vs. Model



# Three-gluon decay of quarkonia $\Upsilon(9.46)$ , $\Upsilon(10.02)$



**MD g-jet is Farry**

$$P_n(s) = \sum_{m'=0} \frac{(m'+1)(m'+2)}{2(\bar{m}/3)^2} \left(1 - \frac{1}{\bar{m}/3}\right)^{m'} C_{3+m'}^n N_g \left(\frac{\bar{n}_g^h}{N_g}\right)^n \left(1 - \frac{\bar{n}_g^h}{N_g}\right)^{(3+m')N_g - n}$$

$$m' = m - 3$$

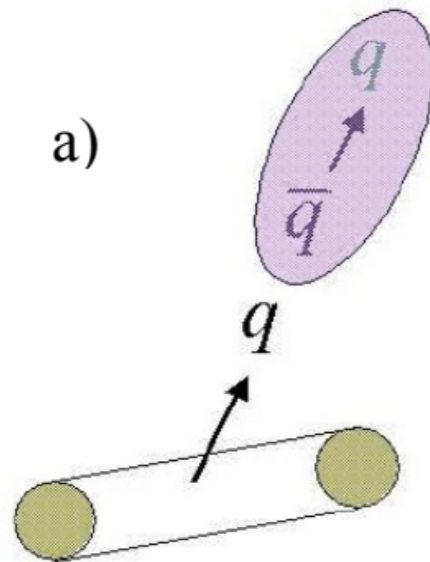
$$\Delta \bar{n} = \bar{n}(\Upsilon \rightarrow 3g) - \bar{n}(e^+e^- \rightarrow q\bar{q})$$

$$\Delta \bar{n}(s)_{exp} \approx \Delta \bar{n}(s)_{theor} \approx 0.8$$

# pp interactions

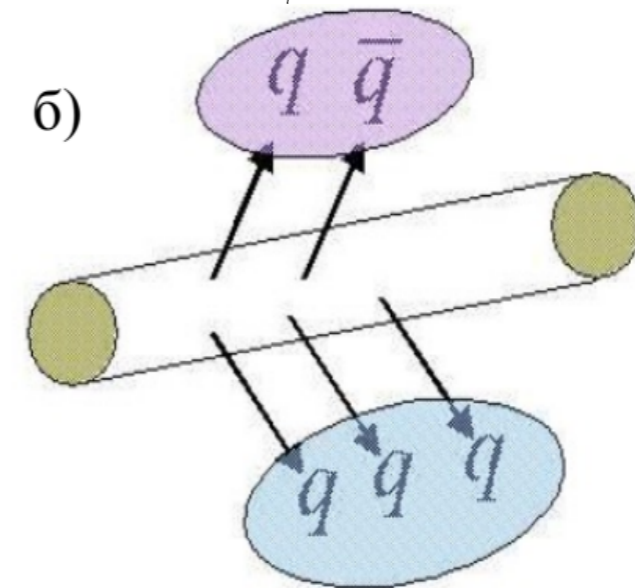
- Applying same procedure led to smaller hadronization parameters as in  $e^-e^+$
- Decreasing number of valence quark, parameters start grow
- Gluon Dominance Model (GDM)

**Fragmentation  
(vacuum)**



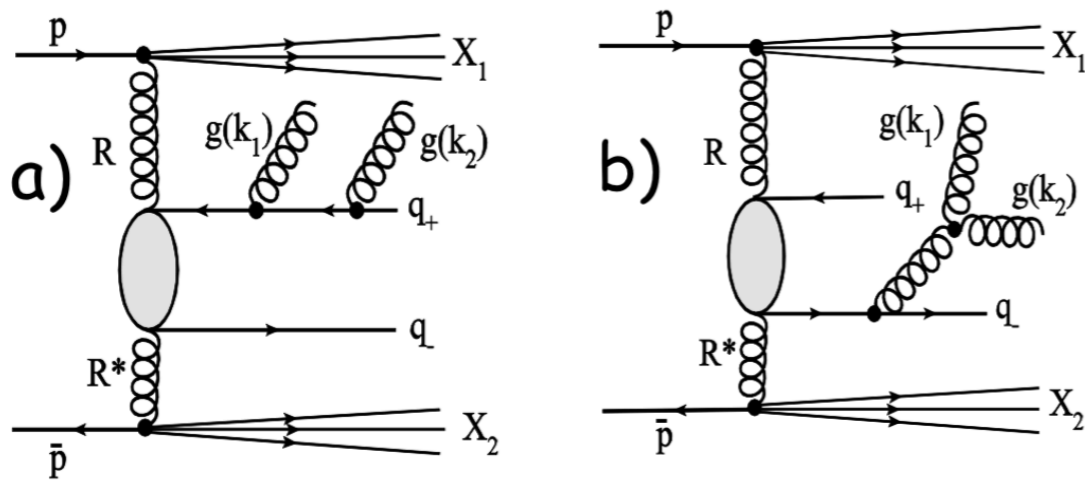
$$R = \frac{N_B}{N_{\pi^0}} \ll 1$$

**Recombination  
(quark-gluon system)**



$$R \approx 1$$

# pp interactions

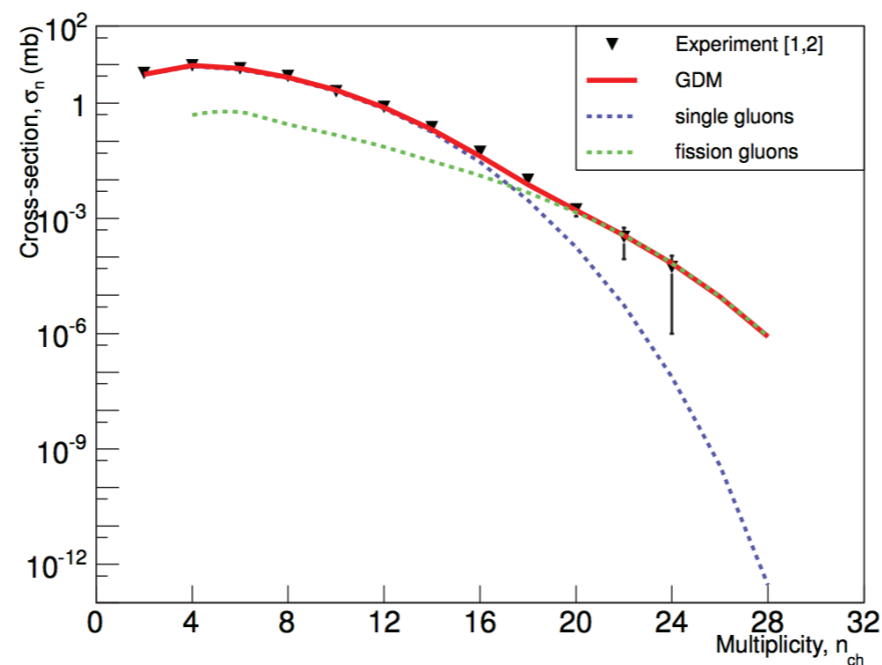


**At HM region** formation of two gluon jets predominates in the case b) in comparison with the case a).

## Superposition of 2 distributions

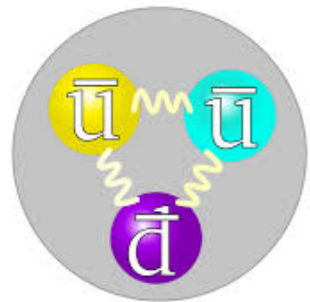
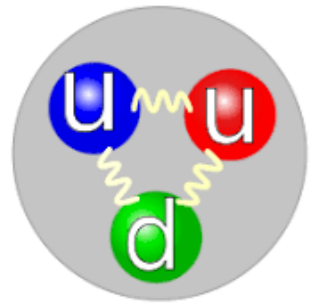
$$P_n(s) = \alpha_1 \sum_{m_1=1}^{Mg_1} \frac{\bar{m}_1^{m_1} e^{-\bar{m}_1}}{m_1!} C_{m_1 N}^{n-2} \left( \frac{\bar{n}^h}{N} \right)^{n-2} \left( 1 - \frac{\bar{n}^h}{N} \right)^{m_1 N - (n-2)} + \alpha_2 \sum_{m_2=1}^{Mg_2} \frac{(2\bar{m}_2)^{m_2} e^{-2\bar{m}_2}}{m_2!} C_{m_2 N}^{n-2} \left( \frac{\bar{n}^h}{N} \right)^{n-2} \left( 1 - \frac{\bar{n}^h}{N} \right)^{m_2 N - (n-2)}$$

**single gluons**  
(without branching)



**fission gluons**  
(with branching)

# p(anti)p annihilation

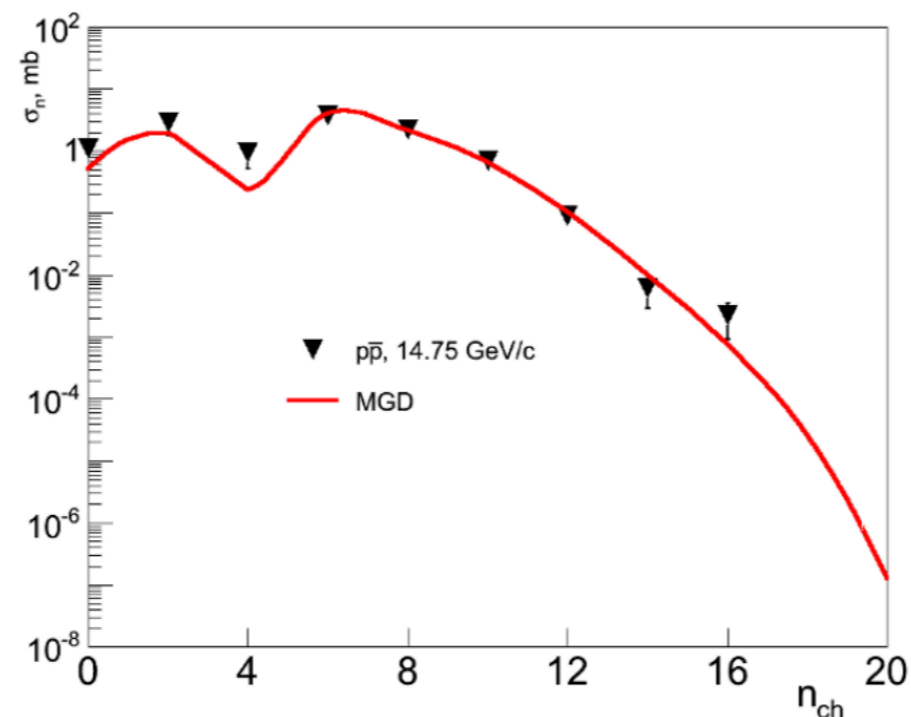


“0” topology  $\rightarrow 3 \pi^0$

“2” topology  $\rightarrow \pi^0, \pi^-, \pi^+$

“4” topology  $\rightarrow \pi^+, \pi^+, \pi^-, \pi^-$

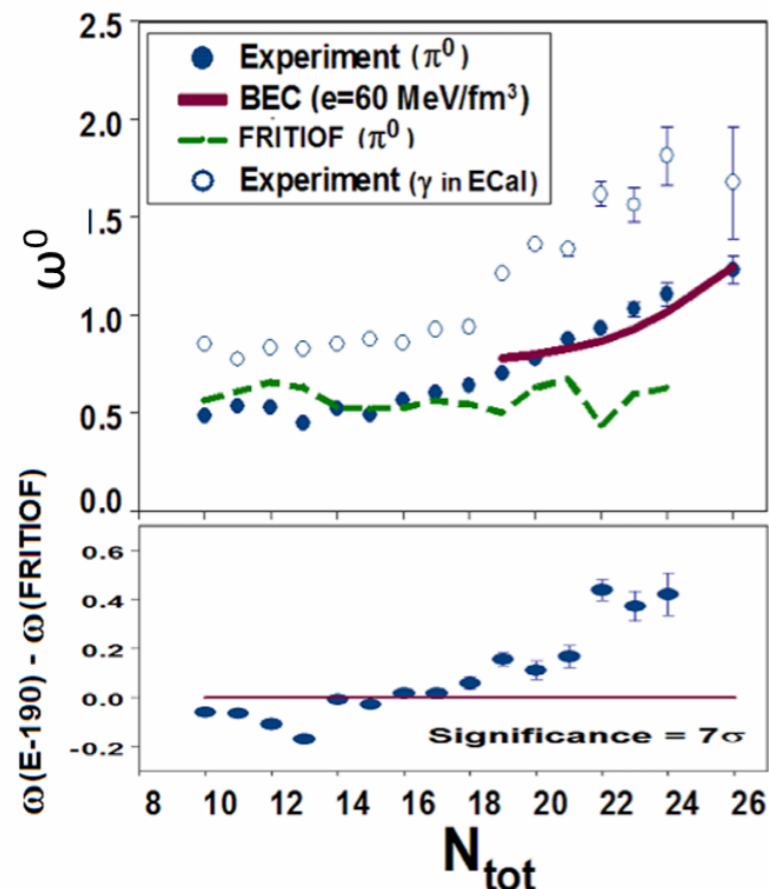
$$G(z) = c_0 \sum_m^{M_0} P_m^G \left[ 1 + \frac{\bar{n}^h}{N}(z - 1) \right]^{mN} + c_2 z^2 \sum_m^{M_2} P_m^G \left[ 1 + \frac{\bar{n}^h}{N}(z - 1) \right]^{mN} + c_4 z^4 \sum_m^{M_4} P_m^G \left[ 1 + \frac{\bar{n}^h}{N}(z - 1) \right]^{mN}$$





# Conclusion

- description of MD in  $e^+ e^-$  and  $p(\text{anti})p$  annihilation,  $pp$  interactions and 3-gluon decay of  $\Upsilon$  introducing the hadronization scheme in a wide energy region -> using mathematical approaches of probability theory
- fitting experimental data with GDM using ROOT packages
- Protvino: SVD2 setup - for the first time SVD Collaboration got the evidence to the pionic Bose-Einstein condensate formation in HM region



# **Thank you for your attention**

Special thanks to:  
Organisers  
&  
Dr. Elena Sergeevna Kokoulina